Improper Integrals — extra problems

1. Let $f(x) = \dfrac{\sqrt{4x^8 + x^2 - 3x + 1} + \ln(x) - 1}{x^2 + \sin^2 x - 5}$. Find a function $g(x)$ so that $f(x) \sim g(x)$, and so that you are able to determine if $\displaystyle\int_1^\infty g(x)\,dx$ converges. Use this to determine if $\displaystyle\int_1^\infty f(x)\,dx$ converges.

2. Does the integral $\displaystyle\int_1^\infty \frac{\sin^4 x}{4x^3 + x + 3}\,dx$ converge or diverge?

3. Does the integral $\displaystyle\int_0^\infty \frac{\arctan x}{2 + e^x}\,dx$ converge or diverge?

4. Does the integral $\displaystyle\int_0^\infty \frac{(e^x + \ln(x) + 1)^3 - \sqrt{x} - 5}{2 + x^3 + e^{2x}}\,dx$ converge or diverge?

Compute the integral:

5. $\displaystyle\int_0^9 \frac{1}{\sqrt[3]{x - 1}}\,dx$

6. $\displaystyle\int_{-1}^2 \frac{x}{(x + 1)^2}\,dx$